

AN064

Representing Deformable Mirror Influence Functions with Zernike Polynomials

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1 Introduction

Commonly available optical modeling software like Code 5 and ZEMAX Optic Studio use Zernike polynomials to represent optical wavefronts. Modelers often want to use Zernike polynomials native to these codes to model deformable mirror (DM) influence functions (IFs) because it is convenient. In this application note we study how many influence functions it takes to adequately represent a plate-type DM IF.

2 Modeling

The modeling used for this study was setup in Matlab and leveraged the both the MZA basic Matlab library and the AOS Propagation and AdaptiveOptics classes.

2.1 Setup

Figure 1 shows the adaptive optics (AO) overlay used in the modeling. We generated the influence functions for this deformable mirror (DM) with 97 actuators using plate theory. Since there was significant symmetry, we chose to only fit the influence functions from the center to the bottom edge in this evaluation. These were influence functions with indices 49 to 54. The DM was designed to have a diameter equal to 110mm. The actuator spacing was set to 10mm. Each of the influence function amplitudes was normalized prior to fitting to make the results easier to interpret. We then used a least-squares technique to fit the DM influence function to Zernike polynomials with the Zernike radius equal to 100mm. We varied the number of Zernikes used in the fit. The Zernike ordering used was the n-l ordering from Malacara's book.[1] For each fit, we evaluated the peak-to-valley (PV) and root-mean-square (RMS) of the residual inside the Zernike circle. The code for this study is shown in the appendix.

[1] D. Malacara, Optical Shop Testing, © 2007 John Wiley & Sons, Inc.

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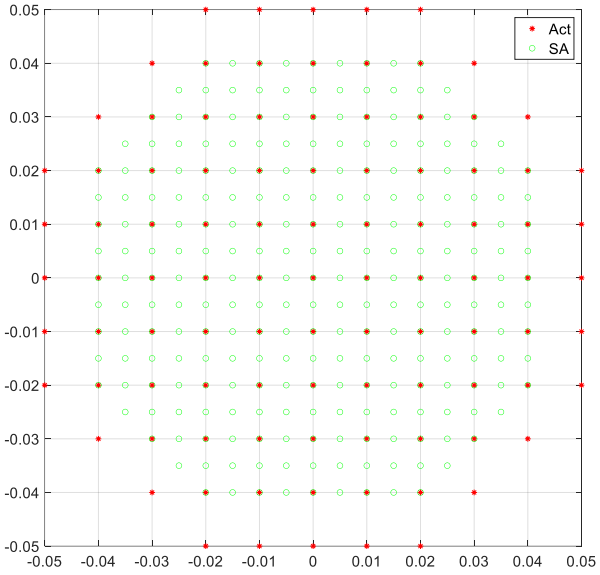
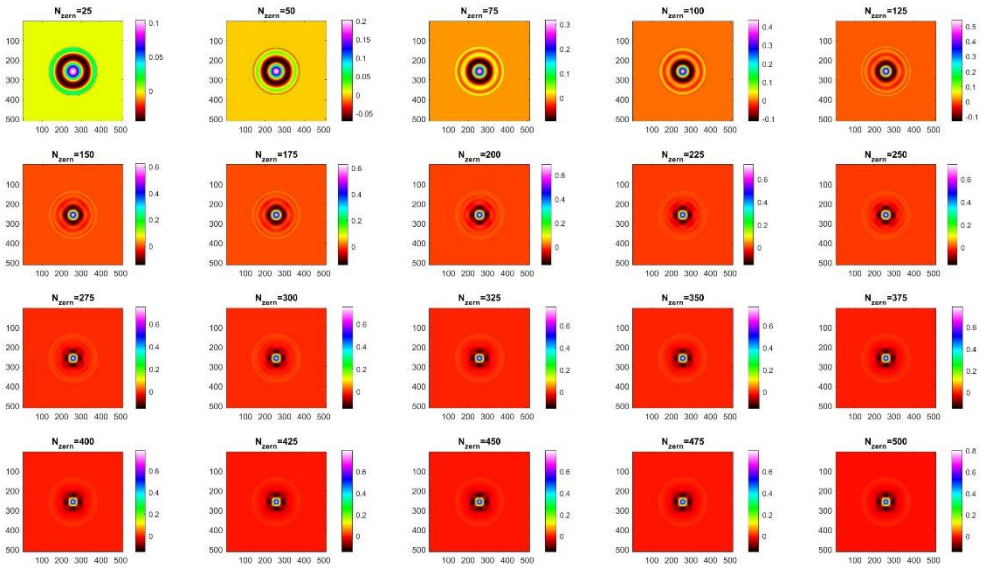


Figure 1: Adaptive Optics Deformable Miror Actuator (Act) and Sub-Aperture (SA) Position Overlay

2.2 Results

Figure 2 shows the fit for each of the actuators from 49 (central) to 54 (edge).



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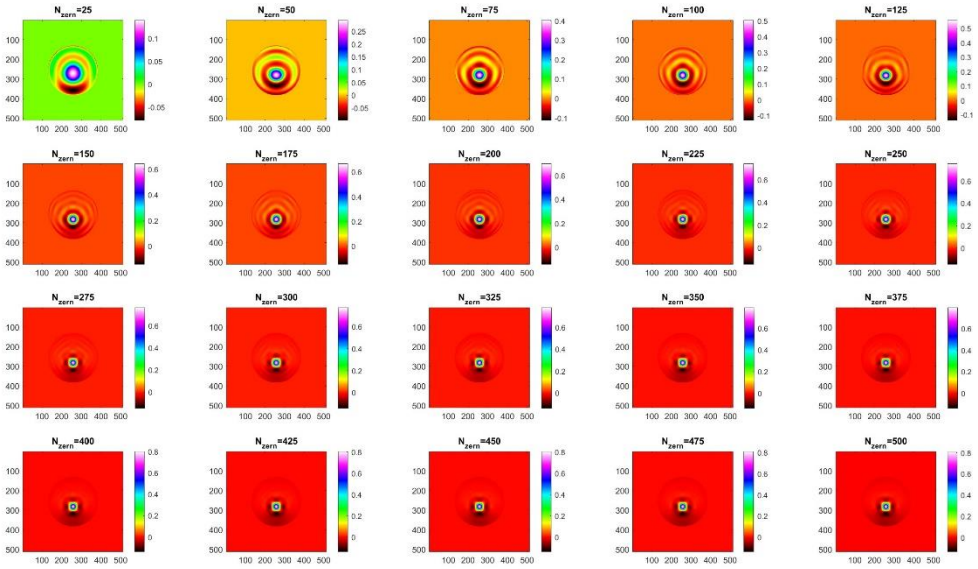

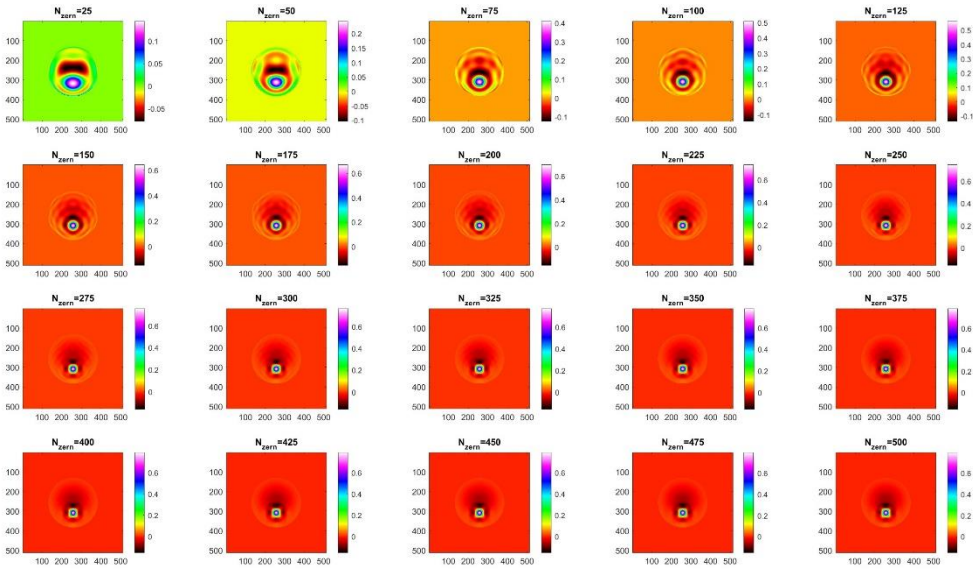

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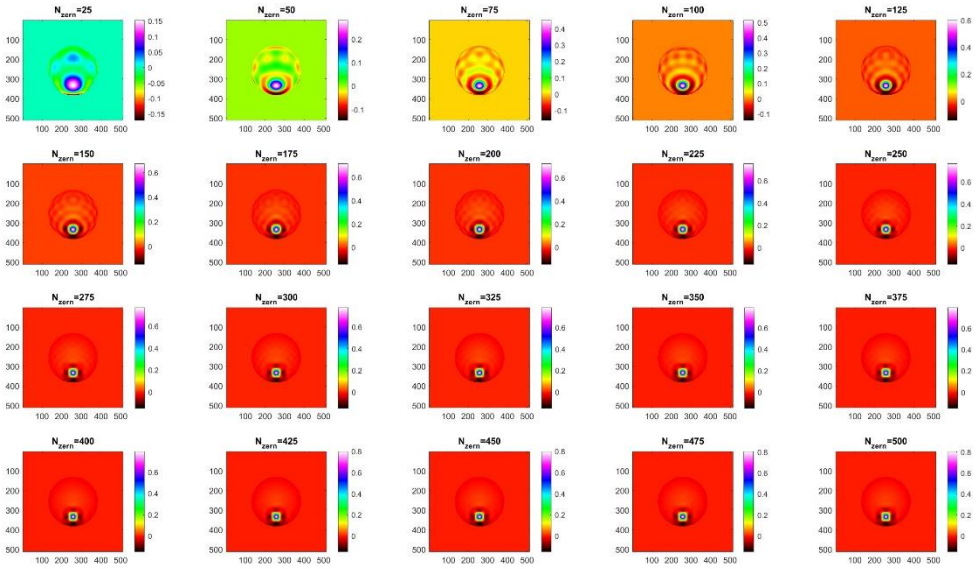

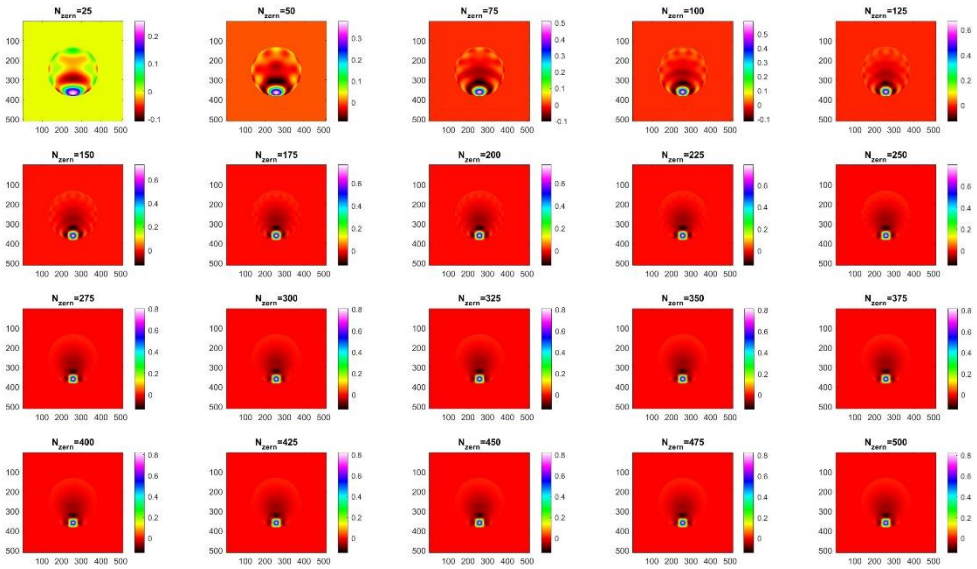

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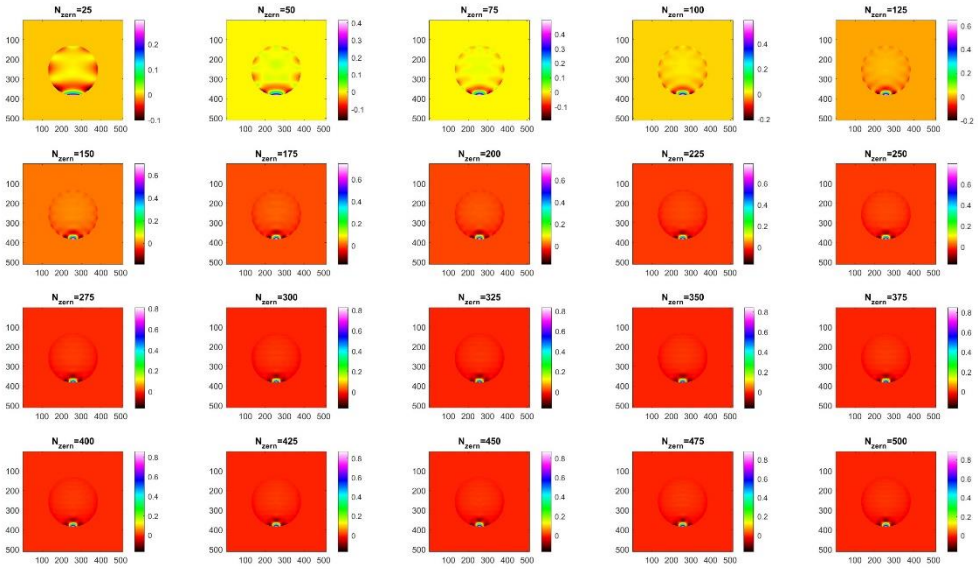

Figure 2: Fitting Results for Actuators 49 (top) through 54 (bottom) for Varying Number of Zernike Terms Used in the Fit

Figure 3 shows the RMS and peak-to-valley (PV) residual to the fit for varying number of Zernike terms in the fit. It is easy to see that the PV residual trends toward one as the number of Zernikes used in the fit decreases due to the actuator influence function normalization. The PV does not get below 10% for all actuators until around 375 Zernikes are used. We only studied up to 500 Zernike terms so the PV never reached below 1%. The RMS results are better numerically but never get below 0.1% fit error up to 500 Zernikes. Visually, the fits start to look good around 300 Zernikes.

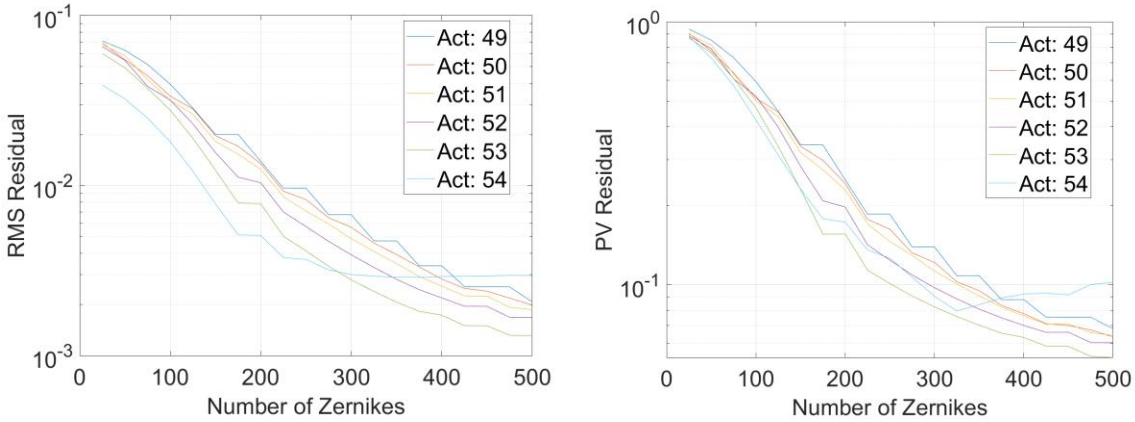

Figure 3: Fit Residual Amplitude in RMS (left) and Peak-To-Valley (right)

3 Conclusions

In this study, we found that the number of Zernikes required to adequately represent a plate-type DM influence function is at least several hundred. At 500 Zernike terms, the PV residual never dropped below 1%. In general, we recommend not using Zernike terms for modeling DMs.

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4 Appendix: Modeling Code

```
% Aberration Compensation Example
close all; clear all; clc; format compact;
farfield = 1;
ppt = 1;
dbgAO = 1;
ss = get(0, 'Screensize');

%% parameters
Dap = 100e-3;
Nact1D = 10;
dact = Dap ./ Nact1D;
dsub = dact/2;
dxsub = dsub/2;
dysub = dsub/2;
wavelength = 0.9e-6;
leak = 0.9999; gain = 0.1;
IAmax = 6e-6; PVmax = 16e-6;
p = Propagation;
a = AdaptiveOptics;

%% calculated parameters
z = Dap.^2 / 4 / wavelength / 20
k = 2*pi/wavelength;

%% create aberration
h = figure;
set(gcf,'Position',ss + [100 100 -200 -200]);

nxy = 512; %1024;
dxy = 2*Dap/nxy;
g = p.makeGrid(nxy,dxy);
v = zeros(1,15); v(12)=wavelength;
phs = ZernikeCompose2(v,nxy,dxy,Dap/2);
phs = phs .* (g.r<Dap/2);
figure(h); clf; show(g.x,g.y,phs); hold on; plotCircle(Dap,0,0,'w--');
title('Phase');
phsVec = phs(:); phsVec = phsVec(phsVec~=0);
RMSwfeInput = std(phsVec(:))

%% Setup propagation
g = p.makeGrid(nxy,dxy);
ap = (g.r<Dap/2.*.99);
% Dspot = 2*f*wavelength/Dsa
% Dsa/Dspot = 2 --> Dspot = Dsa/2
% Dsa = 4*f*wavelength/Dsa
% f = Dsa^2 / wavelength / 4
efl = dsub^2 / wavelength / 4 % set for spot diameter = Dsa/2

%% Setup AO
a.SetupSHWFS(dxy,efl,wavelength);
a.SetGrid(nxy,dxy);
a.MakeActGrid(dact,Dap+dact);
a.MakeSubapertureGrid(dsub,Dap-2*dsub,dxsub,dysub);
```


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```
% a.SetIFType(1); % Gaussian
a.SetIFType(2); % Plate
a.MakeIFs();  % change to plate influence functions...
Nact = length(a.xact)
dact_pitch = a.dact

%% AO Overlay Plot
figure(h); clf; plot(a.xact,a.yact,'r*');
hold on;
plot(a.xsub,a.ysub,'go');
axis image;
grid on; drawnow;
legend('Act','SA');
if (ppt) ToPPT2('Actuator Overlay'); end;

%% analyze IF stroke
NzernVec = [25:25:500];
Nact = length(a.IF)
rZern = Dap/2;
NactVec = find((abs(a.xact)<1e-5) .* (a.yact>-1e-5)), %[1 10 round(Nact/2)];
NactVec = NactVec(:);
c=0;

%% force normalization
for ii=1:length(NactVec); % 1:Nact;
    idx = NactVec(ii)
    PV0(ii) = PV(a.IF{idx}.*ap);
    a.IF{idx} = a.IF{idx}./PV0(ii);
end;

%% scan and fit
for Nzern = NzernVec;
    c=c+1
    c2=0;
    for ii=1:length(NactVec); % 1:Nact;
        idx = NactVec(ii)
        c2=c2+1;
        IF = a.IF{idx};
        [v]=ZernikeDecompose(IF,dxy,dxy,Nzern,rZern);
        [fit]=ZernikeCompose2(v,nxy,dxy,rZern);
        resid = IF-fit;
        fitSave{c,c2} = fit;
        residSave{c,c2} = resid;
        PV1(c,c2) = nonZeroPV(resid.*ap);
        RMS1(c,c2) = NonZeroRMS(resid.*ap);

        clf;
        subplot(1,3,1); show(IF.*ap); title('DM IF');
        subplot(1,3,2); show(fit.*ap); title(sprintf('Zernike Fit %i,%i',idx,Nzern));
        subplot(1,3,3); show(resid.*ap);
        title1 = sprintf('Residual PV=%.2e RMS=%.2e',PV1(c,c2),RMS1(c,c2))
        title(title1);
        drawnow;

    end;
```


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```
end;
close all;
save('ZernikeIFFitResults.mat');

%% final plotting
nf;
subplot(1,2,1); imagesc(NactVec,NzernVec,PV1); title('PV'); colorbar;
subplot(1,2,2); imagesc(NactVec,NzernVec,RMS1); title('RMS'); colorbar;
drawnow;


%% plot fitting
nf([1 1]);
for ii=1:length(NactVec); % 1:Nact;
    clf;
    for jj = 1:length(NzernVec);
        subplot(4,5,jj);
        show(fitSave{jj,ii}.*ap);
        title(sprintf('N_z_e_r_n=%i',NzernVec(jj)));
    end;
    if (ppt)
        ToPPT2(sprintf('IF %i',NactVec(ii)));
    end;
end;

%% zernike fit plots
Nzerns = length(NzernVec)
c2=0;
nf([.8 .8]);
for ii=1:length(NactVec); % 1:Nact;
    idx = NactVec(ii)
    c2=c2+1;
    c=0;
    clf;
    for Nzern = NzernVec;
        c=c+1
        subplot(4,5,c); show(g.x,g.y,fitSave{c,c2}.*ap);
title(sprintf('N_z_e_r_n=%i',Nzern));
    end;
    drawnow;
    pause(1.0);
    if (ppt) ToPPT2(sprintf('IF %i',idx)); end;

    c=0;
    clf;
    for Nzern = NzernVec;
        c=c+1
        subplot(4,5,c); show(g.x,g.y,(fitSave{c,c2} - a.IF{idx}).*ap);
title(sprintf('N_z_e_r_n=%i',Nzern));
    end;
    drawnow;
    pause(1.0);
    if (ppt) ToPPT2(sprintf('IF %i Residual',idx)); end;

    clf;
```


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```
    plot(NzernVec,PV1(:,c2),'bo-'); hold on;
    plot(NzernVec,RMS1(:,c2),'r*-');
    grid on;
    legend('PV','RMS','Location','northeast');
    fontsize(gcf,24,"points");
    drawnow;
    pause(1.0);
    if (ppt) ToPPT2(sprintf('IF %i Residual Error',idx)); end;

    clf;
    semilogy(NzernVec,PV1(:,c2),'bo-'); hold on;
    semilogy(NzernVec,RMS1(:,c2),'r*-');
    grid on;
    legend('PV','RMS','Location','northeast');
    fontsize(gcf,24,"points");
    drawnow;
    pause(1.0);
    if (ppt) ToPPT2(sprintf('IF %i Residual Error',idx)); end;

end;

%% final summary
clear leg;
for ii=1:length(NactVec);
    leg{ii}=sprintf('Act: %i',NactVec(ii));
end;
nf([1064         380         831         616]);
semilogy(NzernVec,PV1); grid on;
ylabel('PV Residual'); xlabel('Number of Zernikes');
legend(leg,'Location','Best');
fontsize(gcf,24,"points");
if (ppt) ToPPT2(); end;

nf([1064         380         831         616]);
semilogy(NzernVec,RMS1); grid on;
ylabel('RMS Residual'); xlabel('Number of Zernikes');
legend(leg,'Location','Best');
fontsize(gcf,24,"points");
if (ppt) ToPPT2(); end;
```


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