ACTIVE OPTICAL SYSTEMS

AN060

Spherical Aberration Amplitude Relative to Parabolic Shape

Author: Dr. Justin Mansell
Revision: 6/11/23

1 Introduction

While it is easy to create a spherical optic using conventional polishing techniques, parabolic optics induce a lower amplitude of aberration on the reflected beam providing higher performance. The difference between the sphere and the parabolic shapes is primarily the Zernike term spherical aberration, but also contains higher even order radial terms. Optical manufacturers often want to know the amplitude of the deviation from sphere to discern which manufacturing techniques and how much time would be required to create the optic.

2 Analytical Derivation

The shape of an ideal parabolic optic is given by,

$$z(x) = \frac{x^2}{2R},$$

where x is the lateral radius and R is the optic radius of curvature. The variable x was chosen to represent the lateral radius due to the potential for confusion with the radius of curvature.

The shape of a spherical surface is given by,

$$z(x) = \sqrt{R^2 - x^2} \approx R \cdot \left(1 - \frac{x^2}{2R^2} + \frac{x^4}{8R^4} - \cdots\right),$$

where the variable definitions are the same and the approximation is the Taylor series expansion of the spherical shape. The largest difference between the Taylor series and the parabolic shape is the 4th order term in the Taylor series. Figure 1 shows a comparison of the spherical and parabolic shapes and a comparison of the difference between them and the 4th order term from the Taylor series for a 30cm diameter optic with a 1.2m radius of curvature.

© Active Optical Systems, LLC

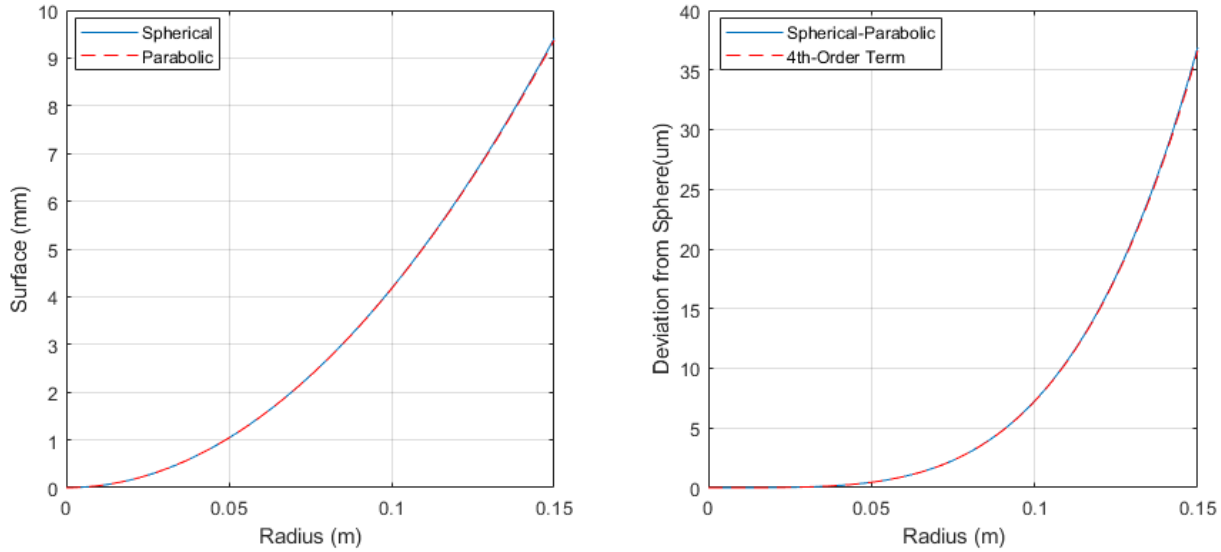

Figure 1: (left) Comparison of Spherical and Parabolic Surfaces; (right) Spherical to Parabolic Difference and Taylor Series 4th Order Term

The 4th order term alone produces a very large amplitude at the edge, so it is usually balanced against additional parabolic term when manufacturing. To do this, we can use the equation,

$$\Delta z(x) = \frac{x^4}{8R^3} - \frac{x^2}{2R_{eff}},$$

where R_{eff} is some effective radius that will balance the 4th and 2nd order terms. We can balance them by setting the edge deviation at the maximum radius to zero. The effective radius of curvature is then given by,

$$R_{eff} = 4 \cdot \frac{R^3}{x_{max}^2},$$

where x_{max} is the optic maximum radius.

If we take the derivative of the equation for Δz and find the local maxima, we can solve for the maximum deviation, which happens at one third of the maximum optic radius (x_{max}). The resulting amplitude of deviation from sphere is given by,

$$\Delta z_{max} = \frac{x_{max}^4}{32 \cdot R^3}.$$

Figure 2 shows a comparison of the simple spherical to parabolic deviation and the derived spherical aberration formula. The deviation amplitude formula produced a value of 9.1 microns, which corresponds well with the graphical results.

We can also represent this in another form that may be easier to use and remember. If the optic diameter is double x_{max} and the effective focal length from a collimated input beam is the radius of curvature over two, then the optic f/# is given by f/D or R/2 / (2*x_{max}) or R/(4*x_{max}). The f/# raised to the -3rd power, is then given by,

$$(f/\#)^{-3} = \frac{64 \cdot x_{max}^3}{R^3}.$$

Therefore the amplitude of the deviation from sphere is represented by,

$$\Delta z_{max} = \frac{x_{max}}{32}\left(\frac{1}{64 \cdot (f/\#)^3}\right) = \frac{x_{max}}{2048 \cdot (f/\#^3)}.$$

Using the 0.15m diameter and 0.6m focal length, we get the 9.1-micron deviation again using this formula.

© Active Optical Systems, LLC

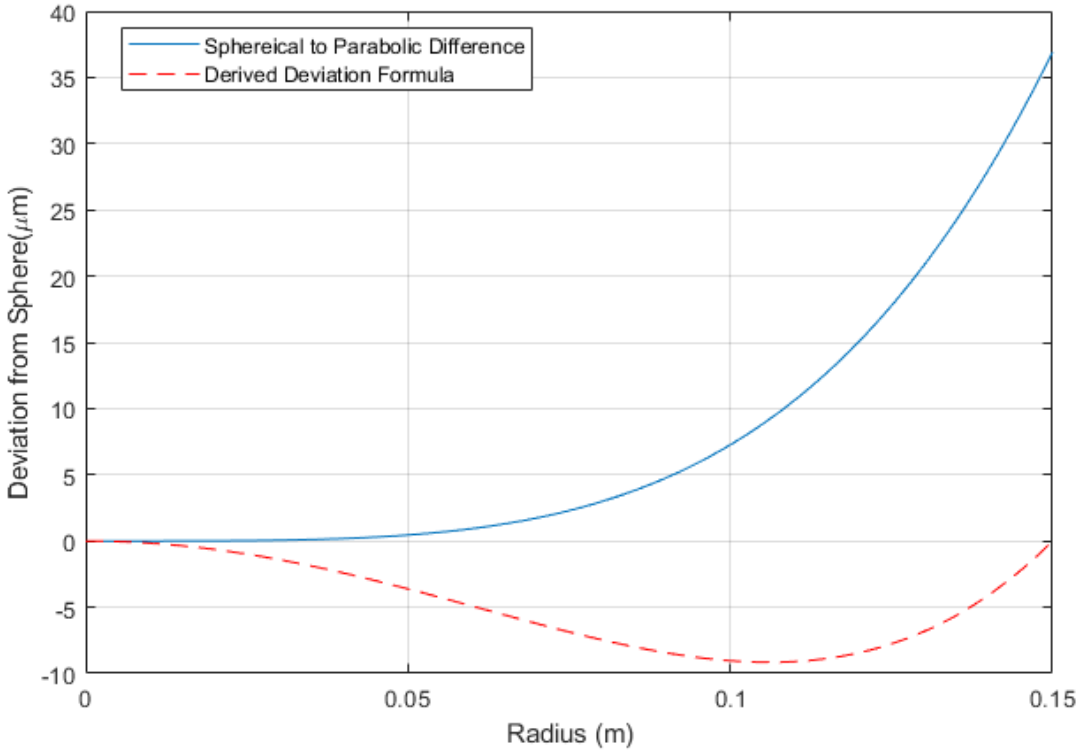

Figure 2: Spherical to Parabolic Difference and Derived Spherical Aberration Formula

3 Conclusions

In this application note, we derived a formula for the amplitude of the deviation of a spherical shape from parabola as the maximum radius over 2048 divide by the optic f/# raised to the third power.

© Active Optical Systems, LLC

4 Appendix: Matlab Test Code

```
%
Setup;
ppt=0;

R=1.2;
xmax = 30e-2 / 2;

x = 0:xmax/100:xmax;
R1 = R;% * 1.004;

z1 = R1 - sqrt(R1.^2 - x.^2);
z2 = x.^2 ./ (2*R);
dza = x.^4 ./ (8*R.^3);

nf([.8 .8]);
subplot(1,2,1); plot(x,z1*1e3); hold on; plot(x,z2*1e3,'r--'); grid on;
xlabel('Radius (m)');
ylabel('Surface (mm)');
legend('Spherical','Parabolic','Location','Best');
subplot(1,2,2); plot(x,(z1-z2)*1e6);  hold on; plot(x,1e6*dza,'r--'); grid on;
xlabel('Radius (m)');
ylabel('Deviation from Sphere(um)');
legend('Spherical-Parabolic','4th-Order Term','Location','Best');

%% find formula
%dza2 = (x.^4 ./ (8*R.^3)) - (x.^2 ./ (2*R/0.0035));
Reff = 4*R^3/(xmax^2)
dza2 = (x.^4 ./ (8*R.^3)) - (x.^2 ./ (2*(Reff)));
dza2 = (1/(8*R^3)) .* (x.^4 - x.^2*xmax^2);

dzmax = xmax^4/(32*R^3)

nf([.8 .8]);
plot(x,(z1-z2)*1e6);  hold on; plot(x,1e6*dza2,'r--'); grid on;
xlabel('Radius (m)');
ylabel('Deviation from Sphere(um)');
```


© Active Optical Systems, LLC