ACTIVE OPTICAL SYSTEMS

AN059

Ray Matrix Equation for Telescope Focus Distance

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1 Introduction

A telescope made from two simple ideal lenses can be easily analyzed using ray matrices to achieve an equation for the focus of the telescope as a function of lens separation distance. While we will be analyzing this as if it were lenses, the same equations and approach can be used for reflective telescopes as well. This application note focusses on the amount of separation change required to change the telescope focus.

2 Ray Matrix Approach

To analyze this problem, we assume a collimated input beam into a two lens telescope, like that shown in Figure 1.

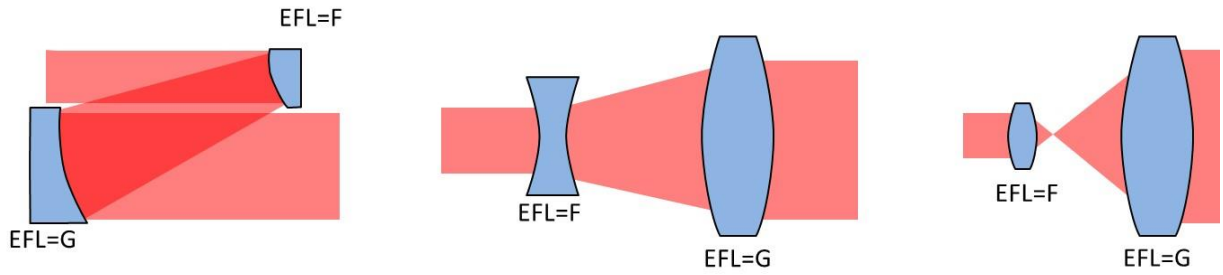

Figure 1: Example Telescope Designs Relevant to this Solution

The ray matrices required for input to output of this system are given by,

$$M = \begin{bmatrix} 1 & 0 \\ -1/G & 1 \end{bmatrix}\begin{bmatrix} 1 & D \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/F & 1 \end{bmatrix} = \begin{bmatrix} 1 - D/G & D \\ \frac{D}{FG} - 1/G - 1/F & 1 - D/F \end{bmatrix},$$

where F and G are the lens focal lengths and D is the separation distance. The effective focal length is given by,

$$EFL = -\frac{A}{C} = F\frac{D - G}{D - F - G}$$

If D is equal to F + G + Δ, then the focal length is simplified to,

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$$EFL = F\frac{F+\Delta}{\Delta} = \frac{F^2}{\Delta} + \mathrm{F}$$

Reverse solving this for the offset (Δ) as a function of desired effective focal length (EFL), we get,

$$\Delta = \frac{F^2}{EFL - F}$$

To calculate the range of motion required to move between two different focal distances (EFL1 and EFL2), we get the expression,

$$\delta\Delta = \frac{F^2(EFL1-EFL2)}{(F-EFL1)(F-EFL2)}.$$

If EFL1 and EFL2 are much greater than F, this can be simplified to,

$$\delta\Delta = \frac{F^2(EFL1-EFL2)}{EFL1*EFL2},$$

which looks similar to the refractive index reflection coefficient formula in field space.

3 Example Results

In our example, we had a -6 magnification telescope with a separation of the optics by 150mm. This required a 180mm and -30mm EFL lens pair. We used Matlab to solve this with ray matrices and with the analytical formulas for focal length above. Figure 2 shows the results in the 5km to 500m range. The Matlab script for this is below.

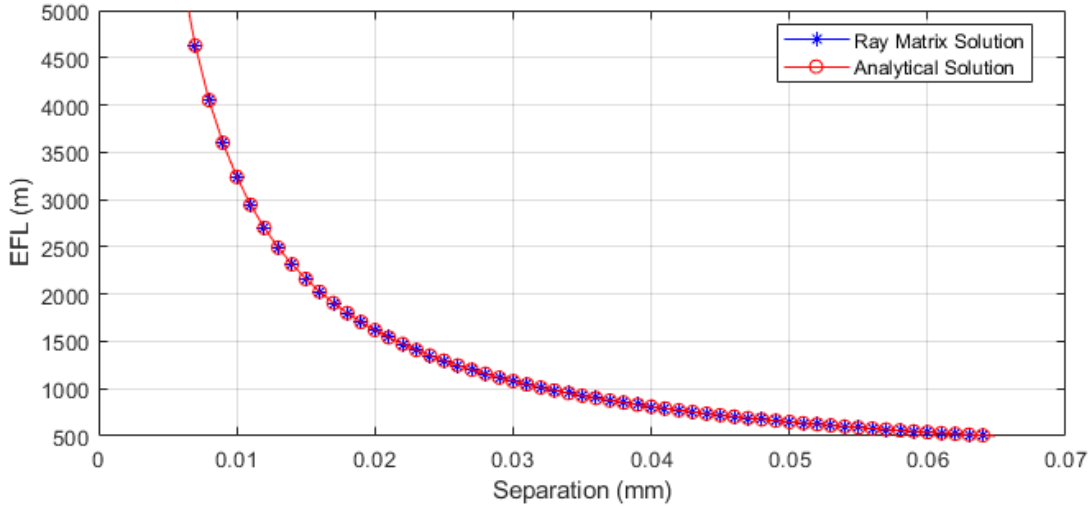

Figure 2 : Effective Focal Length vs. Optica Separation Delta

We also calculated the deltas required for the 5km and 500m focal ranges using the formulas above. They were 6.5µm and 65µm respectively. These results compared favorably to the to the graph above. The δΔ equation matched with the numerical solution as well.

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4 Appendix: Matlab Test Code

```
%
Setup;

d1 = 150
Mag = -1*150/25
% f1+f2 = d1; f2=f1/M; f2 = d1-f1; f1/M = d1-f1; f1(1+1/M)=d1 ;
% f1 = d1/(1+1/M)
f1 = d1 / (1 + 1/Mag)
f2 = f1/Mag

c=0;
deltaVec = [-2:0.001:-0.001 0.001:0.001:2];
deltaVec = [0.001:0.001:0.07];
for delta = deltaVec;
    c=c+1;
    EFL2(c) = (f1^2)./delta + f1;

    M = Lens(f1)*Prop(d1+delta)*Lens(f2);
    A = M(1,1);
    C = M(2,1);
    EFL(c) = -A/C;
end;
nf;
% semilogy(deltaVec,EFL./1000,'b*-'); grid on;
plot(deltaVec,EFL./1000,'b*-'); grid on;
hold on;
plot(deltaVec,EFL2./1000,'r--'); grid on;
xlabel('Separation (mm)');
ylabel('EFL (m)');
ax=axis; ax(3) = 500; ax(4)=5000; axis(ax);


%% solve for delta
EFL1 = 500000; EFL2 = 5000000;
delta1 = f1.^2 / (EFL1-f1)
delta2 = f1.^2 / (EFL2-f1)
```


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