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AN057

Lorentzian-Inspired Marechal Replacement

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1 Introduction

The Marechal approximation is a well-known simple way to relate RMS wavefront error (σ) to the ratio of the on-axis irradiance between an ideal beam and the aberrated beam. It is well known that the Marechal approximation is really only valid for small amplitude RMS wavefront error (σ). This has not stopped countless scaling law simulations from using this approximation even when the wavefront error is far beyond the applicable zone. The parabolic form ($1-\sigma^2/2$) has a clear point of departure from reality as it crosses into negative results when the amplitude is high. In the Gaussian form ($\exp(-\sigma^2)$), this still does not significantly affect the absolute result. Unfortunately, these results often gets applied as a ratio relating the improvement of a beam control system above the performance without it, which makes the ratio artificially high.

In this application note, we explore the use of a modified form of a Lorentizian to not only provide an alternative to the Marechal approximation for the on-axis Strehl ratio, but also to address other key performance metrics like peak Strehl ratio and fractional power in a bucket for differing bucket radii.

2 Modeling Approach

We modeled the focused irradiance on target when a single Kolmogorov-spectrum random phase screens was placed at different amplitudes in the aperture of the system. All this modeling was done using Fourier or wave-optics. We generated 100 random realizations using the Fourier technique and scaled the RMS amplitude to a set of discrete values up to 10 radians in amplitude. Once we had determined the far-field irradiance on target, we calculated five common metrics:

1. On-Axis Strehl Ratio
2. Peak Strehl Ratio
3. Fractional Power in a 1xDL Bucket
4. Fractional Power in a 1.5xDL Bucket
5. Fractional Power in a 2xDL Bucket

The 1xDL bucket was a bucket with a radius equal to the 1.22 times the focal length of the system times the wavelength divided by the broadcast aperture diameter ($1.22\, f\, \lambda/D$). We then averaged together the results to establish a relationship between the metric and the RMS wavefront error.

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In examining the results, especially on a log scale, we saw that the results resembled a Lorentizan. To that end, we proceeded with a fit of the data to the equation,

$$Metric = \left| \frac{1}{b + c|\sigma|^d} \right|$$

where σ is the RMS wavefront error and b, c, and d are all fit coefficients. The fit was accomplished by minimizing the sum of the difference between the average metric data minus the fit divided by the average metric data, or

$$Error = \sum_{n=1}^{N} \frac{y_n - fit_n}{y_n}$$

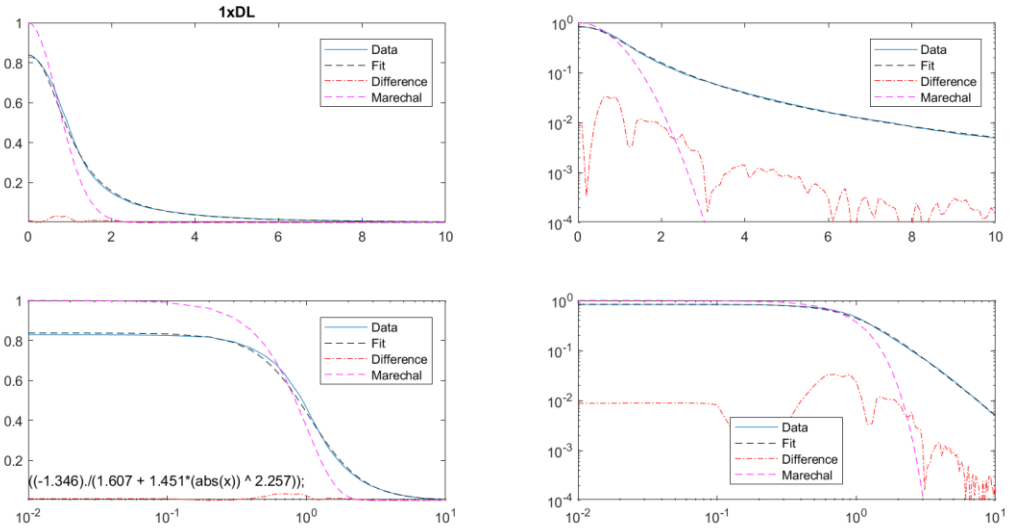
where y_n are the metric points, fit_n is the fit points, and N is the number of samples. The results are summarized in Table 1.

Table 1 : Summary of the Fit Coefficients

Metric	b	c	d
1xDL FPIB	-1.1941442	-1.0777556	2.257
1.5xDL FPIB	1.11539209	0.411222531	2.286
2xDL FPIB	1.068928736	0.254190199	2.308
Peak SR	1.041625249	0.434817513	2.123
On-Axis SR	0.937273787	3.276145628	2.249

2.1 Summary of Results

The following figures summarize the overlap between the model-derived data, the Gaussian version of the Marechal approximation, the fit to the data, and the difference between the data and the fit. The lowest fidelity fit was clearly the On-Axis Strehl Ratio that performed significantly better than Marechal beyond about 2.5 radians RMS WFE, but Marechal might still be better in the lower amplitude. The other fits were significantly better and are advised at all times above Marechal.



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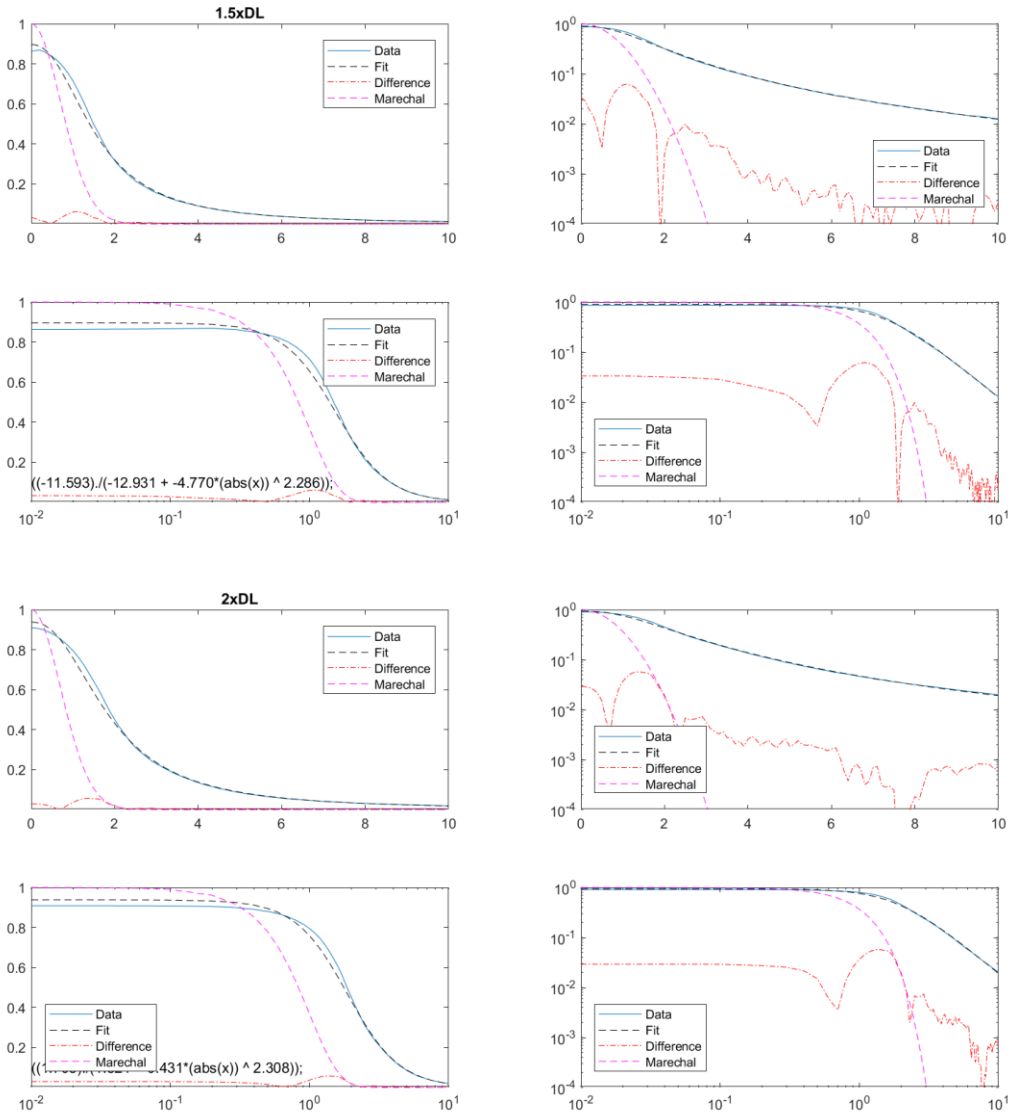

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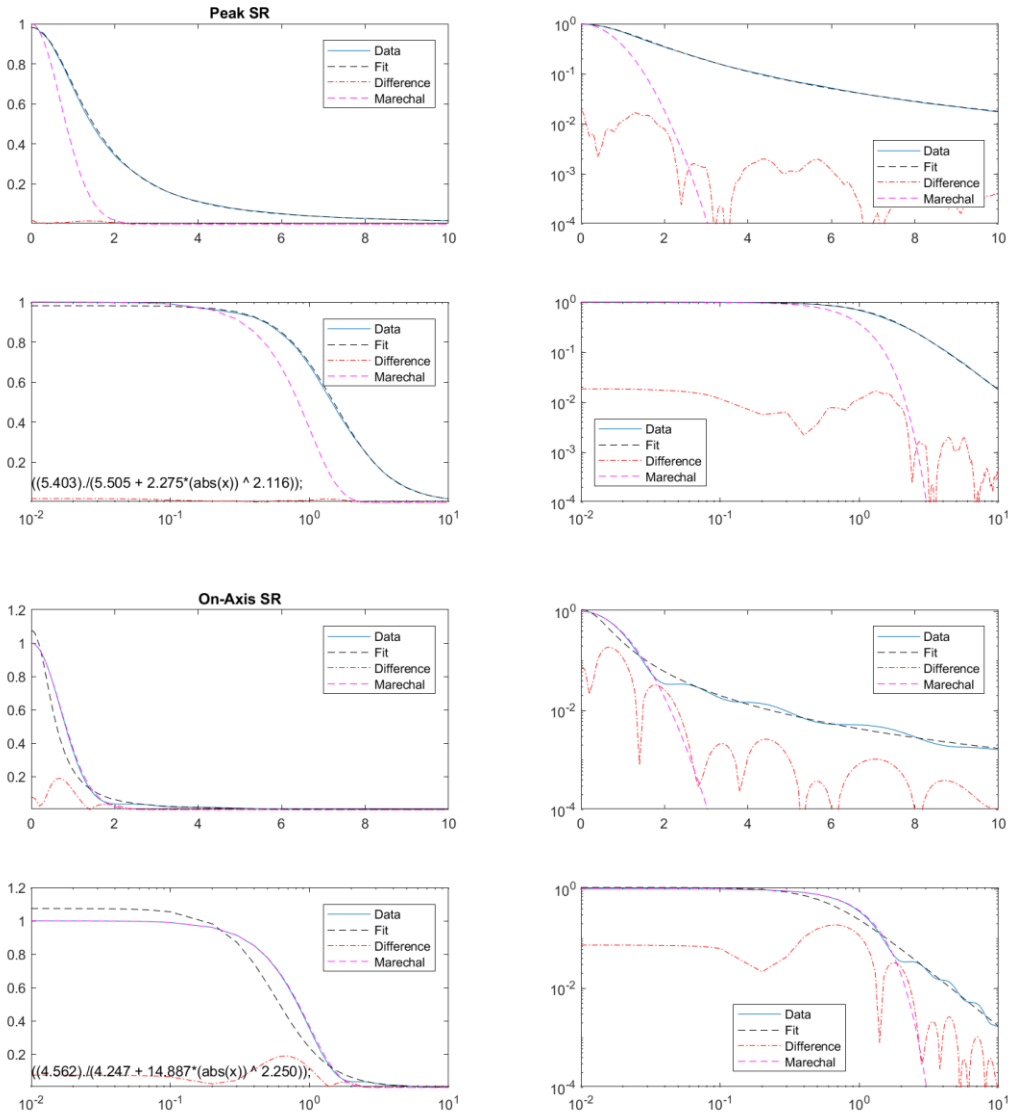

3 Conclusions

In this application note, we derived an alternative for the Marechal approximation based on the Lorentzian equation that much better follows the shape of the wave-optics simulation results using Kolmogorov-spectrum phase screens in the tail of the curve beyond about 2.5 radians RMS wavefront error. Out of this we also found good quality relationships for other key metrics like peak Strehl ratio and various bucket size fractional power in a bucket.

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