ACTIVE OPTICAL SYSTEMS

AN051

The C_n^2 Measurement Limits for a Horizontal Path with a Shack-Hartmann Wavefront Sensor

Author: Dr. Justin Mansell
Revision: 6/9/20

1 Introduction

Several of our customers are using our Shack-Hartmann wavefront sensors to measure atmospheric parameters like C_n^2. As turbulence gets too great, the local wavefront slope can exceed the simple dynamic range of a Shack-Hartmann wavefront sensor. In this application note, we use Komogorov theory to establish the C_n^2 measurement limits using a Shack-Hartmann wavefront sensor without any slope dynamic range extension.

2 Approach and Results

I began by analyzing Fourier-generated Kolmogorov-spectrum phase screens to determine the ratio of the peak to RMS slope magnitude. I generated a set of Kolmogorov phase screens, calculated the gradient of the phase screens and established the phase slope magnitude from the calculated gradients. Then I calculated the RMS and peak amplitude for the 512x512 sample screens. Each screen had a nearly identical 7.5x ratio between the peak and RMS slope amplitudes, so I used this value for my analysis here. We also studied the PV to RMS ratio for the phase screens and found an average over 10 screens of 5.6±0.5.

For a horizontal path, Fried's coherence length for a spherical wave is given by,[1]

$$r_0 = 3.0\left(C_n^2 L k^2\right)^{-3/5},$$

where C_n^2 is the index of refraction variance, r0 is Fried's coherence length, L is the path length, k is the wave number. The 2D tilt variance for a single aperture is given by,

$$\alpha^2_{2D} = 0.364\left(\frac{D}{r_0}\right)^{5/3}\left(\frac{\lambda}{D}\right)^2,$$

where D is the aperture diameter and λ is the wavelength. The RMS and peak tilt can be estimated by,

$$\alpha_{RMS} = \sqrt{\alpha^2_{2D}}$$
$$\alpha_{peak} = 7.5 \cdot \alpha_{RMS}.$$

Combining these we get a simplified equation for peak tilt from the fundamental parameters given by,

[1] R. Tyson, "Principles of Adaptive Optics, 4th Edition".

© Active Optical Systems, LLC

$$\alpha_{peak} = 7.5\sqrt{0.364\left(\frac{D}{3.0\left(C_n^{\,2}Lk^2\right)^{-3/5}}\right)^{5/3}\left(\frac{\lambda}{D}\right)^2}\,.$$

$$\alpha_{peak} = 11.38\cdot D^{-1/6}\sqrt{C_n^{\,2}L}$$

The simple slope dynamic range for a Shack-Hartmann wavefront sensor (without any dynamic range extension) is given by,

$$\theta_{\max_wfs} = \frac{1}{M}\frac{\frac{D_{sa}}{2} - 1.22\frac{f_{sa}\cdot\lambda}{D_{sa}}}{f_{sa}} = \frac{1}{M}\left(\frac{D_{sa}}{2f_{sa}} - \frac{1.22\lambda}{D_{sa}}\right)$$

where D_{sa} is the diameter of an individual sub-aperture, M is the magnification between the wavefront sensor and the output aperture of the telescope aperture, and f_{sa} is the focal length of the lens array. The diameter of the sub-aperture, D_{sa}, can be scaled to the output aperture of the telescope with the optical system magnification, M, by $D = M * D_{sa}$. Including this into the dynamic range equation, a relationship can be established for the maximum C_n^2 as,

$$\theta_{\max_wfs} = \frac{1}{M}\left(\frac{D/M}{2f_{sa}} - \frac{1.22\lambda}{D/M}\right) = \left(\frac{D}{2M^2 f_{sa}} - \frac{1.22\lambda}{D}\right)$$

$$C_n^{\,2}{}_{\max} = \left(\left(\frac{D^{1/6}}{11.38\sqrt{L}}\right)\left(\frac{D}{2M^2 f} - 1.22\frac{\lambda}{D}\right)\right)^2 .$$

The approximate RMS slope noise floor for a Shack-Hartmann wavefront sensor at the output aperture of the telescope assuming the centroid accuracy is about 1/20th of a pixel is given by,

$$\theta_{\min_wfs} = \frac{1}{M}\frac{1}{20}\frac{d_{pix}}{f_{sa}},$$

where d_{pix} is the diameter of a pixel. The minimum C_n^2 can then be determined using this relationship as,

$$C_n^{\,2}{}_{\min} = 1.93\times10^{-5}\cdot\frac{d_{pix}{}^2 D^{1/3}}{M^2 f_{sa}{}^2 L},$$

if the peak slope is set to the RMS slopes. If we set the RMS slopes to the RMS slope formula, we get a minimum C_n^2 of,

$$C_{n\,min}^2 = \frac{1}{900}\frac{d_{pix}^2 D^{\frac{1}{3}}}{M^2 f_{sa}^2 L}.$$

The ratio of the constant term in these formulas is 7.5^2=57. Although this is a bit of a large range, the minimum detectable level is likely between these two values because of the spatial averaging that can occur making the real detectable limit about $\sqrt{N_{sa}}$ lower than the individual sub-aperture RMS, where N_{sa} is the number of sub-apertures. Furthermore, the WFS is sensing the peak slopes, not the RMS wavefront slopes of the turbulence. Experimental calculations or careful modeling will be needed to determine the true nature of the minimum Cn2, but these formulas form a boundary estimate.

3 Summary

To examine these relationships, we executed a Matlab script to plot the maximum C_n^2 for a system with varying magnification and different propagation lengths for a system with the parameters in Table 1. Figure 1

© Active Optical Systems, LLC

and Figure 2 show the maximum and minimum C_n^2 values respectively for varying magnification and different path lengths.

Table 1: Example System Parameters

Parameter	Value
d_{pix}	7.4 microns
f_{sa}	6.7 mm
D_{sa}	150 microns
λ	635nm
D_{camera}	3.6mm

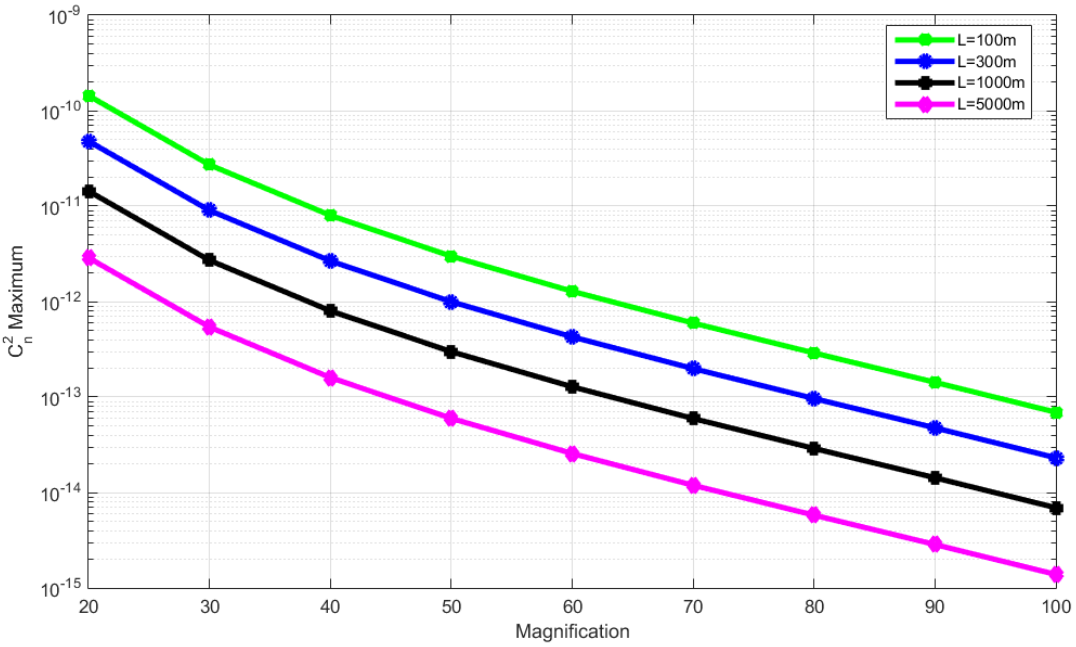

Figure 1: Example Maximum C_n^2 Values

© Active Optical Systems, LLC

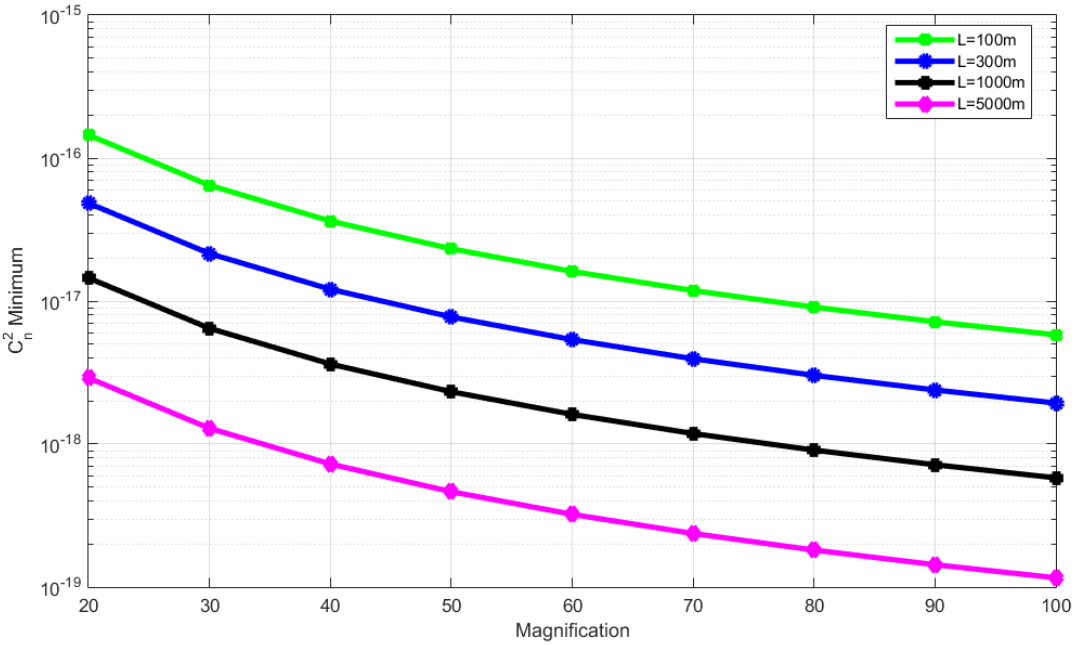

Figure 2: Example System Minimum C_n^2 Values

4 Acknowledgements

Special thanks to Jay Land for pointing out that the square root of the turbulent slope variance should be set to the RMS wavefront slopes instead of using the peak slope formula.

© Active Optical Systems, LLC