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Inter-Actuator Stroke For Kolmogorov Phase Aberrations

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1 Introduction

We can determine the inter-actuator stroke requirements for a deformable mirror (DM) when compensating Kolmogorov-spectrum phase aberrations by analyzing generated phase screens. This will enable us to better specify a DM for compensate aberrations to improve imaging through the atmosphere or improve the performance of a laser communication or weapons system.

2 Analysis & Results

The mean square phase difference between two points separated by r in Kolmogorov-spectrum turbulence is given by,[1]

$$D_\phi = \left\langle \left| \phi(\varepsilon) - \phi(\varepsilon + r) \right|^2 \right\rangle = 6.88 \left(\frac{r}{r_0} \right)^{5/3}$$

where ϕ is the phase and r_0 is the Fried coherence length. This is the average phase difference, but if we want to capture more than the average, this formula is insufficient. To determine the amplitude required for more capture percentage, we analyzed a set of 100 phase screens that were a 16x16 section of a 128x128 point FFT-generated phase screen based on the Kolmogorov-spectrum. This small section is more likely to represent the lowest-order term properly. We then examined the phase difference between every pair of points in the screen and analyzed the distribution of phase differences as a function of point separation distance. We examined the spectrum of phase differences at a separation of one r_0 distance and performed a histogram on the data to determine the frequency of various phase variances. We then performed a cumulative sum on the histogram data to establish a spectrum and normalized the spectrum of phase variances such that the distribution at the 50% frequency is 1.0. Figure 1 shows the extracted scale factors on the square of the phase differences (phase variances) as a function of capture fraction on a 0 to 1 scale. We found that this spectrum was well fit by

$$S(p) = 1.6 \cdot \left(-\ln(1-p) \right)^{1.4}$$

where p is the capture fraction and S is the scale factor.

[1] R. Tyson, "Principles of Adaptive Optics".

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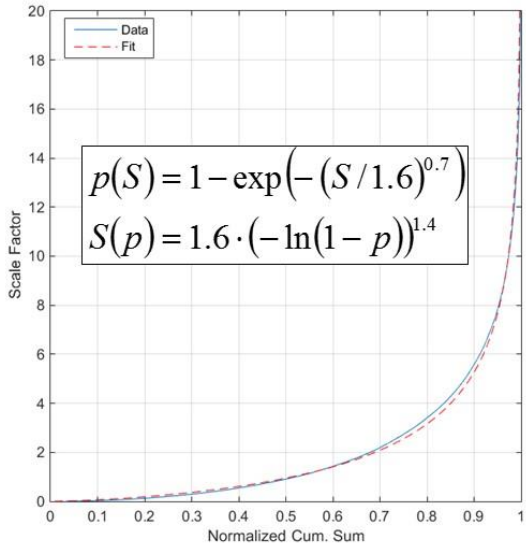

Figure 1: Scale Factor as a function of Cumulated Sum of the Measured Phase Difference Distribution

Based on this analysis, we were able to complete the analysis of the phase difference between two points separated by r_0 as follows:

$$D_\phi = S(p) \cdot 6.88 \left(\frac{r}{r_0} \right)^{5/3}$$

$$r = r_s \approx \text{actuator spacing} = r_0$$

$$\sigma_\phi^{\,2} = S(p) \cdot 6.88$$

$$\sigma_\phi(p = 50\%) \approx \sqrt{6.88 \cdot 1} rads = 2.62 rads \approx 0.42\lambda$$

$$\sigma_\phi(p = 90\%) \approx \sqrt{6.88 \cdot 5.5} rads = 6.18 rads \approx 1\lambda$$

$$\sigma_\phi(p = 95\%) \approx \sqrt{6.88 \cdot 8} rads = 7.4 rads \approx 1.2\lambda$$

This mathematical method applies to points not separated by r_0 as well, but since common practice for compensation of Kolmogorov-spectrum phase aberrations is to separate actuators by r_0 in the outgoing beam space (after the magnification of the beam director).

3 Conclusions

In conclusion, we have shown that if we are separating our actuators in the traditional way by r_0, the phase difference between any two actuators for a 95% solution is only 1.2 waves or 1.2 microns for λ=1.0 microns. Even a 99% capture, the result is only 1.56 waves. Therefore, for nearly any practical case, a DM with 2 microns of phase stroke is sufficient for wavelengths up to 1.3 microns.

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4 Appendix: Matlab Code

```
%
Setup;

ppt=1;

multiplier = 1;
z = 1000;
Dap = 20e-2;
hp = 1;
ht = 1;
wavelength = 1e-6;
Nscreens=1;
nxy = 16;
dseedmax = 200;
dxy = Dap/nxy;
removeTilt = false;
x=((-nxy/2):1:(nxy/2-1)).*dxy;
[xx,yy]=meshgrid(x,x);
rr = sqrt(xx.^2+yy.^2);

% doc Propagation
pr = Propagation();

for dseed=1:dseedmax;
    seed = dseed

[scr]=pr.MakeAtmScreens(z,hp,ht,wavelength,Nscreens,nxy*8,dxy,seed,multiplier
,removeTilt);
    p = scr{1}(1:nxy,1:nxy);
    [r0,sR2,q0,fG,fT,Cn2avg]=pr.CalculateAtmPathParameters(hp, ht, z,
wavelength, Dap, [0 0 0], multiplier);
    r0

    nf; show(p); drawnow; clabel('Radians');
    if (ppt && (dseed<2 || dseed>(dseedmax-2))) ToPPT(); end;

    if (dseed==1)
        N = round(1.42*nxy);
        for i=1:N;
            dpSave{i}=[];
        end;
    end;
    s2 = zeros(1,N);
    cnt = s2;
    tic
    for i=1:nxy;
        disp(sprintf('(%i,%i) - %f s left',i,j,toc/i*(nxy-i)));
        for j=1:nxy;
            idx2 = round(sqrt((xx-xx(i,j)).^2 + (yy-yy(i,j)).^2)./dxy) + 1;
            dp2 = p - p(i,j);
            idx = idx2(:); dp = dp2(:);
            for ii=1:length(idx);
```


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```
                s2(idx(ii)) = s2(idx(ii)) + (dp(ii)).^2;
                dpSave{idx(ii)} = [dpSave{idx(ii)} (dp(ii)).^2];
                cnt(idx(ii)) = cnt(idx(ii)) + 1;
            end;
        end;
    end;

    s2 = s2 ./ cnt; % scale for samples

    %% final display
    r = (0:1:(N-1)).*dxy./r0;
    nf; plot(r,s2);
    hold on;
    plot(r,3*(r).^(5/3));
    xlabel('radius / r_0');
    ylabel('Phase Variance (m^2)');
    if (ppt && (dseed<2 || dseed>(dseedmax-2))) ToPPT(); end;

    %% look at distribution at r=r0;
    [mv,mi]=min(abs(r-1))
    mi = round(r0/dxy)+1
    dp2s = dpSave{mi};
    [y,x]=hist(dp2s,length(dp2s)/20);

    var = sum(dp2s)./length(dp2s)
    varcalc = 6.88 .* ((mi-1)*dxy/r0).^(5/3)

    nf; loglog(x,y);
    hold on;
    loglog(x,300.*x.^-0.5);
    loglog(6.88,1,'k*');
    ylabel('Counts');
    xlabel('Phase Variance');
    if (ppt && (dseed<2 || dseed>(dseedmax-2))) ToPPT(); end;

    nf; plot(x,y);
    hold on;
    plot(x,300.*x.^-0.5);
    plot(6.88,1,'k*');
    ylabel('Counts');
    xlabel('Phase Variance');
    if (ppt && (dseed<2 || dseed>(dseedmax-2))) ToPPT(); end;

    nf; plot(cumsum(y));
    if (ppt && (dseed<2 || dseed>(dseedmax-2))) ToPPT(); end;
    close all;

end;

%% cum sum
ycs = cumsum(y);
nf; plot(x,ycs./max(ycs));
xlabel('value');
ylabel('Normalized Cum. Sum');
grid on;
```


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```
hold on;
tau = 4;
yfit = 1-exp(-1*(x/tau).^0.7);
plot(x,yfit,'r--');
legend('Data','Fit','Location','Best');
[mv,mi]=min(abs(yfit-0.5))
xsf = x ./ x(mi);
if (ppt) ToPPT(); end;

nf; plot(xsf,ycs./max(ycs));
xlabel('Scale Factor');
ylabel('Normalized Cum. Sum');
grid on; hold on;
tau = 1.6;
yfit2 = 1-exp(-1*(xsf/tau).^0.7);
plot(xsf,yfit2,'r--');
axis([0 20 0 1]);
legend('Data','Fit','Location','Best');
if (ppt) ToPPT(); end;

nf; plot(ycs./max(ycs),xsf);
ylabel('Scale Factor');
xlabel('Normalized Cum. Sum');
grid on; hold on;
plot(yfit2,xsf,'r--');
axis([0 1 0 20]);
legend('Data','Fit','Location','Best');
if (ppt) ToPPT(); end;

[mv,mi]=min(abs(yfit-0.5));
SF50p=xsf(mi)
dp50p=sqrt(6.88*SF50p)
dp50p_waves=sqrt(6.88*SF50p)/2/pi
[mv,mi]=min(abs(yfit-0.9));
SF90p=xsf(mi)
dp90p=sqrt(6.88*SF90p)
dp90p_waves=sqrt(6.88*SF90p)/2/pi
[mv,mi]=min(abs(yfit-0.95));
SF95p=xsf(mi)
dp95p=sqrt(6.88*SF95p)
dp95p_waves=sqrt(6.88*SF95p)/2/pi
```


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