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AN048

Power in the Bucket to Strehl Ratio Relationship

Author: Dr. Justin Mansell
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1 Introduction

Strehl Ratio (SR) is an easy way to evaluate the efficacy of a beam control system, especially using the Marechal approximation ($SR \approx \exp\left(-\sigma_{WFE}{}^2\right)$). For delivery of laser power to a target, Strehl Ratio is not the best metric for beam control. In this application note, I explore the relationship between these two metrics for Kolmogorov-spectrum turbulence.

2 Analysis

Table 1 summarizes the modeling parameters. I began by writing a Matlab script (the code is in the appendix) that applied a set of 3 Kolmogorov-spectrum phase screen, each of which was generated using the Fourier method (in the Propagation class script available for AOS customers) with varying phase amplitude, to a flat-top circular intensity profile evenly distributed along the propagation path to the focus. I then propagated the field to a focus and analyzed the second moment, power in the bucket (PIB), and the Strehl ratio. The PIB results were measured for a bucket 1x, 1.5x, and 2x the diffraction limited (DL) spot diameter. The PIB is presented as a relative term with the total power as the denominator. Then I analyzed the relationship between these quantities.

Table 1: Modeling Parameters

Parameter	Value	Notes
Diameter	20cm	Flat-Top Field
Number of Screens	3	Evenly Distributed to the Focus
Distance to Focus	1500m	
Number of Realizations	100	
Wavelength	1μm	
Mesh Samples	256	
Mesh Separation	1.6mm	
HV5/7 Multipliers	[0.0001 1:1:10]	

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3 Results

3.1 Strehl Ratio and PIB to Second Moment

One of the clearest relationship between any two quantities was that between the Strehl Ratio and the second moment. Figure 1 shows the relationship of the second moment in meters for the X, Y, and average of X and Y vs. Strehl Ratio. The relationship is more linearized in Figure 2 where the second moment is plotted as its inverse, again with the second moment is in meters. Finally, Figure 3 shows the nearly linear relationship between the PIB and the square root of the inverse second moment.

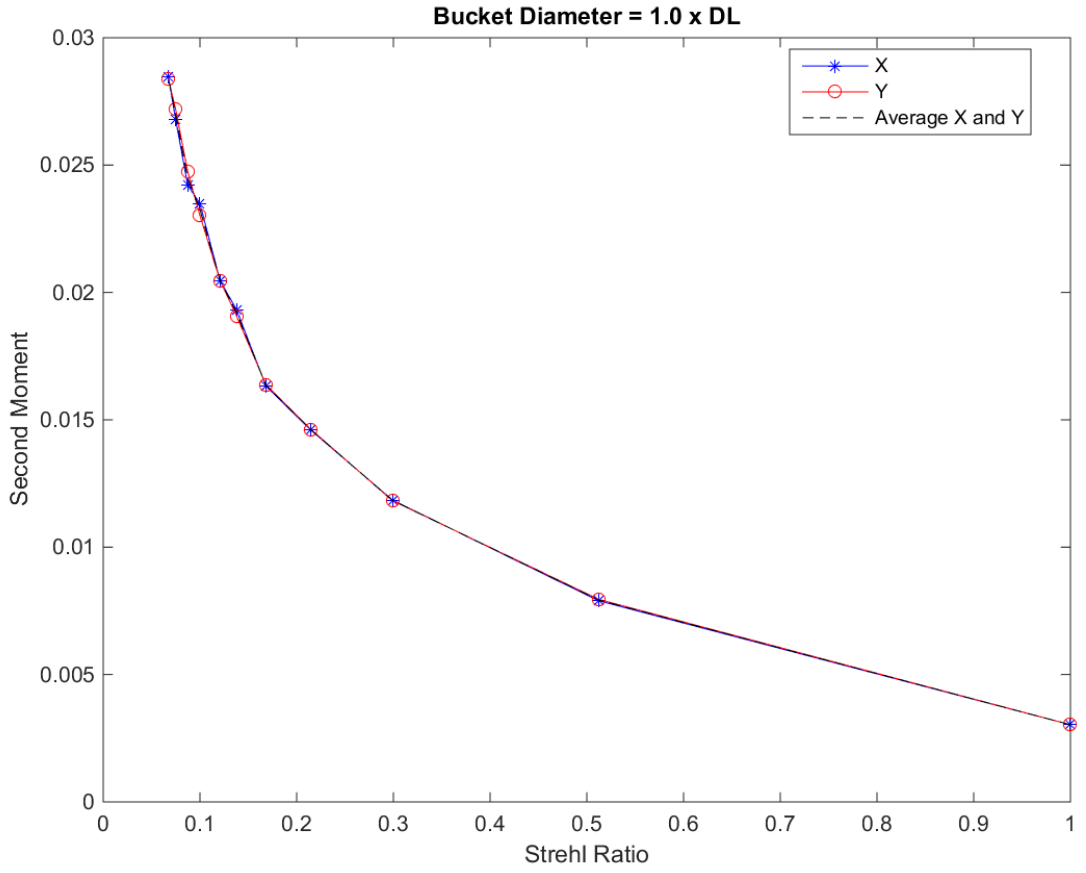

Figure 1: Second Moment (in meters) to Strehl Ratio

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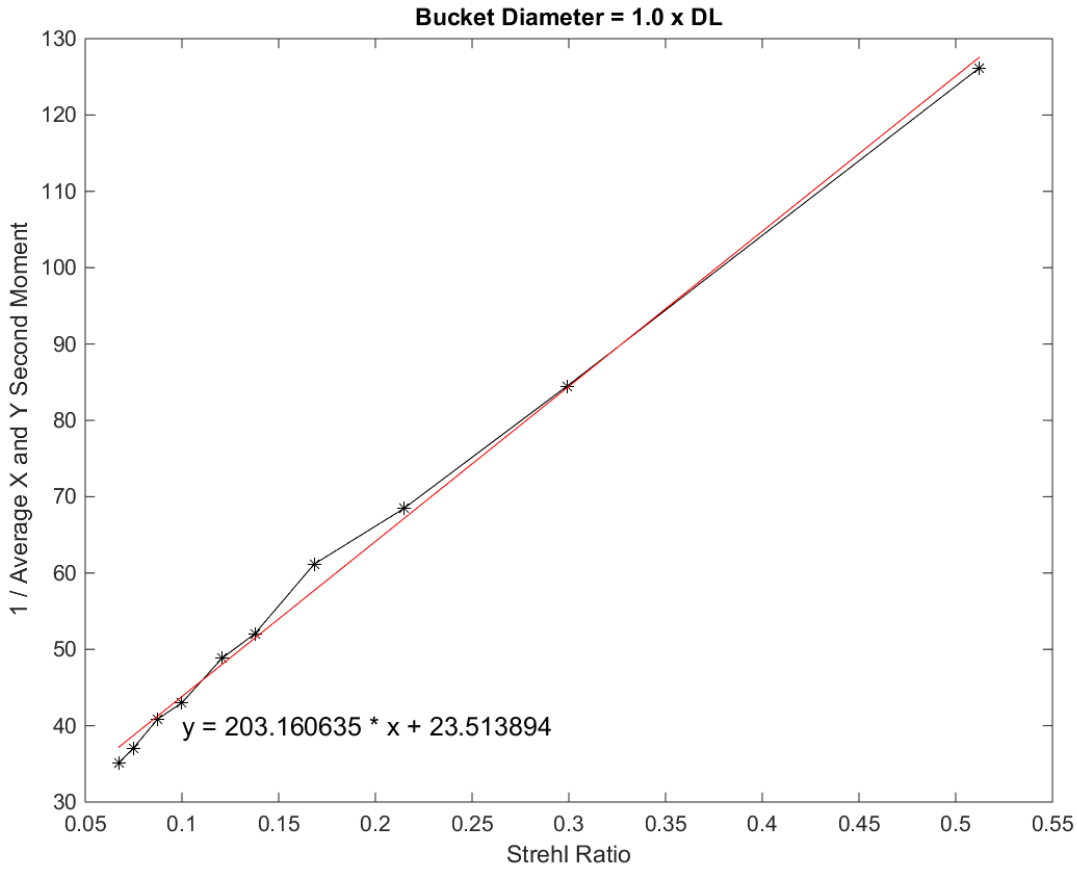

Figure 2: Inverse Second Moment (meters) to Strehl Ratio

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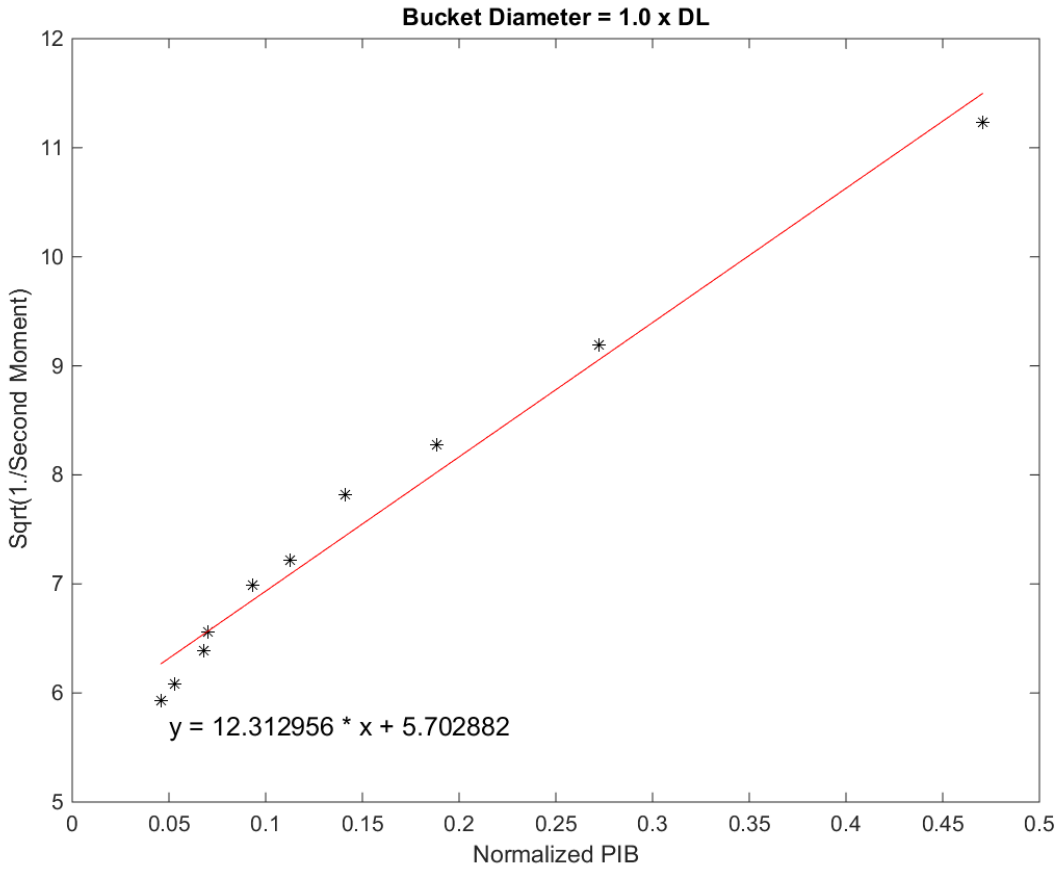

Figure 3: PIB vs. Second Moment (meters)

3.2 Strehl Ratio to 1xDL PIB

Next I analyzed the PIB to the Strehl Ratio for the 1xDL bucket size. Figure 4 shows all the data points that the modeling produced and a linear fit to the data with the fit relationship being PIB = 0.84 * SR + 0.0009.

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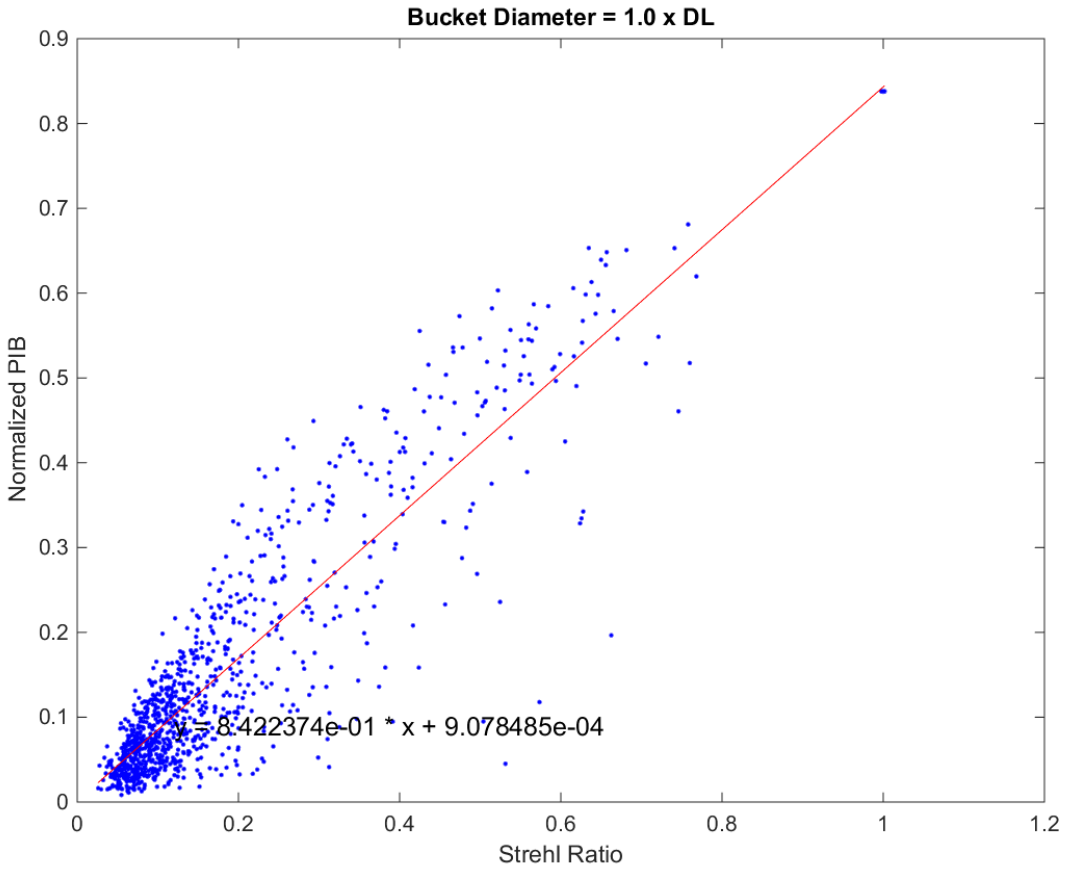

Figure 4: PIB to Strehl Ratio for 1xDL Bucket Size

3.3 Strehl Ratio to 2xDL PIB

Next I analyzed the 2xDL bucket size data and saw a much less linear relationship. Figure 5 shows all the data points and that a linear fit that is not terribly effective. Figure 6 shows a more effective fit to the average data from each turbulence strength. The resulting fit is given by,

$$PIB \approx 0.85 \cdot \left(1 - 1.16 \cdot \exp\left(-SR/0.27\right)\right) + 0.09 .$$

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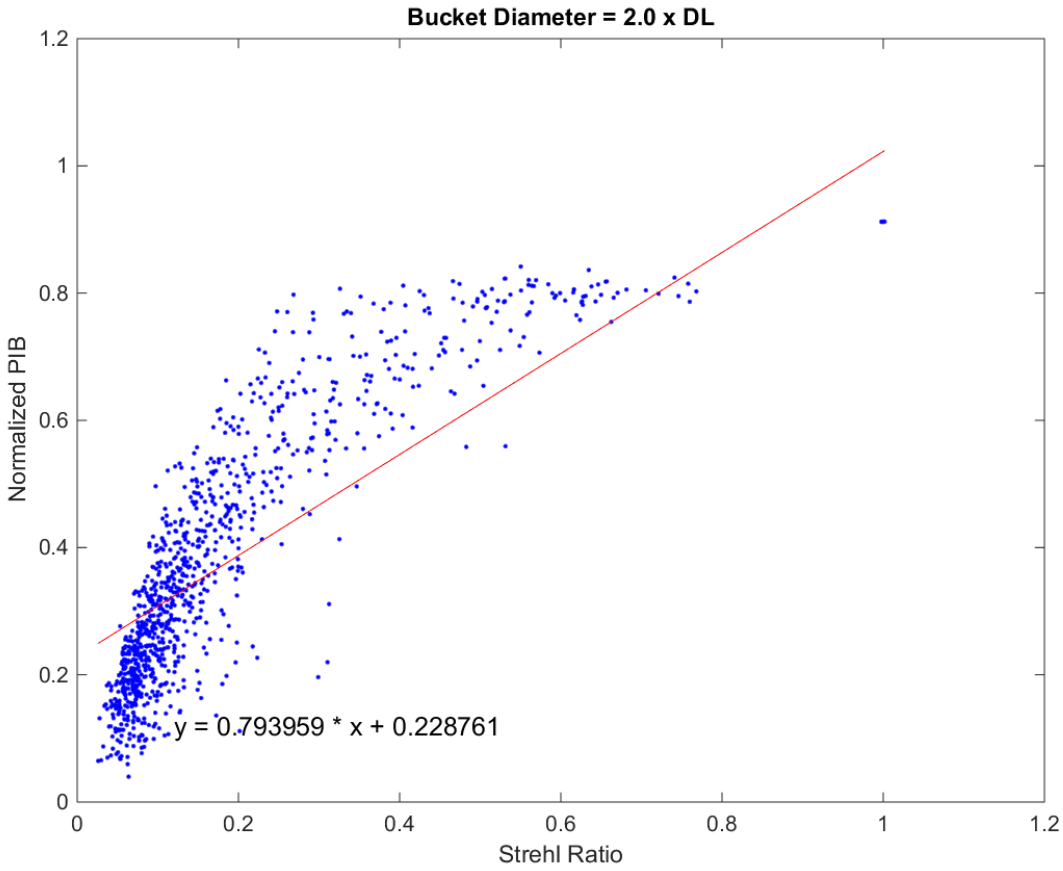

Figure 5: PIB to Strehl Ratio for the 2xDL Bucket Size

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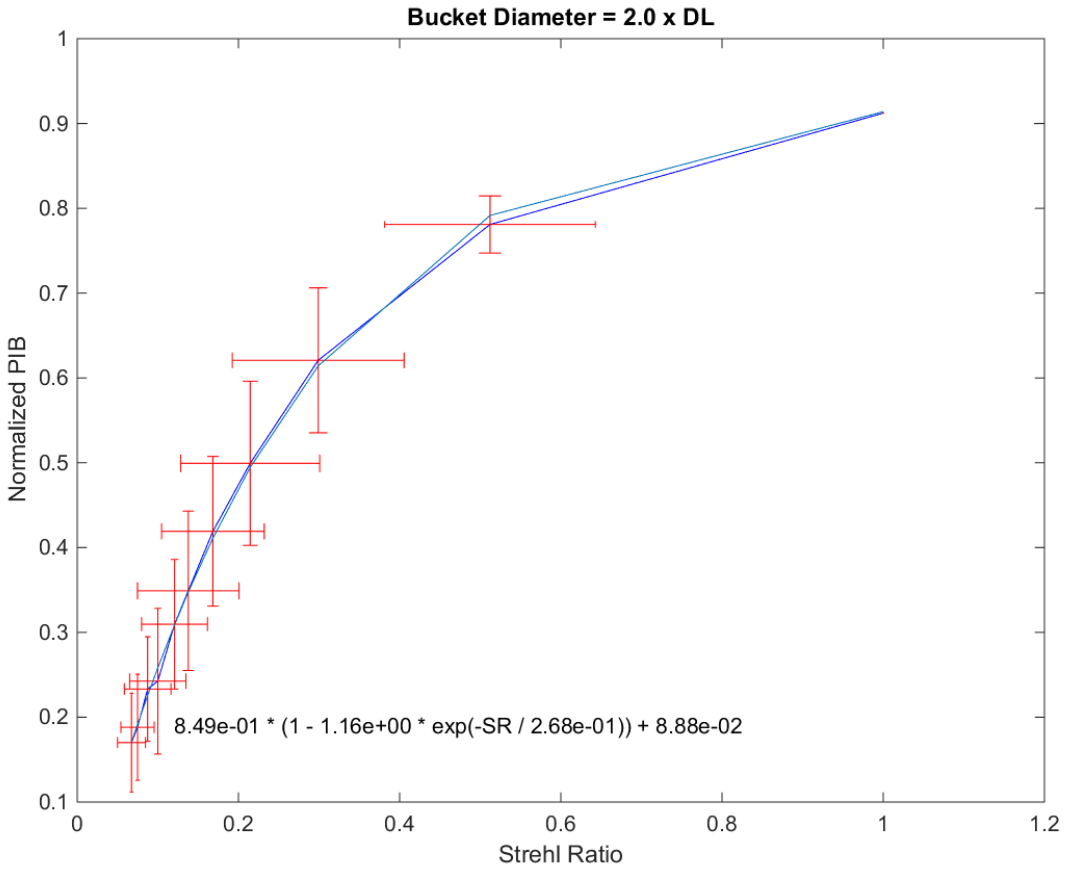

Figure 6: Average PIB for different turbulence strengths vs. Strehl Ratio

4 Conclusions

In conclusion, there are some clear relationships that can be established from this modeling. The second moment is inversely proportional to the Strehl ratio. The PIB is fairly linearly related to the square root of the inverse second moment. Finally, for a diffraction limited bucket size (diameter equal to the diffraction limited spot diameter), the PIB is linearly related to the SR with a 0.84 scale factor and no appreciable offset.

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5 Appendix: Matlab Code

```
%relationship between PIB and Strehl Ratio
Setup;

for NxDL = [1 1.5 2];
    clearAllBut('NxDL');
    ppt = 1;
    Dap = 20e-2;
    seed0 = 12345;
    multiplier = 4;% * 1e-10;
    Nscreens = 3;
    wavelength = 1e-6;
    hp = 10;
    ht = 10;
    z = 1500;
    nxy = 256;
    dxy = Dap*2/nxy;
    p = Propagation;
    rmax = 100; %realization max

    if (ppt) TitleSlidePowerPoint('PIB to Strehl Ratio'); end;

    E = aperture(nxy,dxy,Dap/2,0,0,0) .* p.QPF(nxy,dxy,z,wavelength);
    Dff = NxDL * 2.44 * z * wavelength / Dap;
    bucket = aperture(nxy,dxy,Dff/2,0,0,0);
    [Eout,Esteps] = p.FresnelPropagate(z,E,dxy,wavelength);
    Iideal = abs(Eout).^2;
    denom = max(Iideal(:));
    Ptotal = sum(sum(Iideal));

    nf([.8 .8]); c=0;
    for multiplier = [0.0001 1:10];
        c=c+1;
        for r=1:rmax;
            [phs] =
p.MakeAtmScreens(z,hp,ht,wavelength,Nscreens,nxy,dxy,seed0+r+(rmax+1)*c,multi
plier);
            [Eout,Esteps] = p.StepPropagate(z,E,phs,dxy,wavelength);
            I = abs(Eout).^2;

            SR(r,c) = max(I(:)) ./ denom;
            PIB(r,c) = sum(sum(I.*bucket));
            PIBn(r,c) = sum(sum(I.*bucket))./Ptotal;
            [xc,yc,x2(r,c),y2(r,c)] = MomentsWithThreshold(I,0.01);

            if(r==rmax)
                clf;
                subplot(2,2,1); show(I);
                subplot(2,2,2); show(bucket);
                subplot(2,2,3); show(I .* bucket);
                subplot(2,2,4);
                plot(SR);
                hold on;
                plot(PIBn);
```


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```
            legend('Strehl Ratio','PIB','Location','Best');
            title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
            drawnow;
        end;
    end;
end;

%% summary
SRavg = mean(SR)
SRstd = std(SR)
PIBn_avg = mean(PIBn)
PIBn_std = std(PIBn)
secMomX = mean(x2) * dxy;
secMomY = mean(y2) * dxy;
secMom = (secMomX + secMomY)/2;

%% second moment and PIB plot
nf([.8 .8]);
plot(PIBn_avg,secMomX,'b*-');
hold on;
plot(PIBn_avg,secMomY,'ro-');
plot(PIBn_avg,secMom,'k--');
legend('X','Y','Average X and Y','Location','Best');
xlabel('Normalized PIB');
ylabel('Second Moment');
%LinearFit(gcf,SRavg,PIBn_avg,12,'r-');
title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
drawnow;
if (ppt) ToPPT(); end;

nf([.8 .8]);
plot(PIBn_avg(2:end),(1./secMom(2:end)).^0.5,'k*');
xlabel('Normalized PIB');
ylabel('Sqrt(1./Second Moment)');
LinearFit(gcf,PIBn_avg(2:end),(1./secMom(2:end)).^0.5,12,'r-');
title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
drawnow;
if (ppt) ToPPT(); end;

%% second moment and Strehl Ratio plot
nf([.8 .8]);
plot(SRavg,secMomX,'b*-');
hold on;
plot(SRavg,secMomY,'ro-');
plot(SRavg,secMom,'k--');
legend('X','Y','Average X and Y','Location','Best');
xlabel('Strehl Ratio');
ylabel('Second Moment');
%LinearFit(gcf,SRavg,PIBn_avg,12,'r-');
title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
drawnow;
if (ppt) ToPPT(); end;

nf([.8 .8]);
PWR = 1;
plot(SRavg(2:(length(SRavg))),1./(secMom(2:(length(SRavg))).^PWR),'k*-');
```


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```
    xlabel('Strehl Ratio');
    ylabel('1 / Average X and Y Second Moment');

LinearFit(gcf,SRavg(2:(length(SRavg))),1./(secMom(2:(length(SRavg))).^PWR),12
,'r-');
    title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
    drawnow;
    if (ppt) ToPPT(); end;

    nf([.8 .8]);
    plot(secMom,PIBn_avg,'b*-');
    legend('X','Y','Average X and Y','Location','Best');
    xlabel('X and Y Second Moment');
    ylabel('Normalized PIB');
    LinearFit(gcf,secMom,PIBn_avg,12,'r-');
    title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
    drawnow;
    if (ppt) ToPPT(); end;

    %% strehl and PIB plot
    nf([.8 .8]);
    %plot(SRavg,PIBn_avg,'b*--');
    errorbar2(SRavg,PIBn_avg,SRstd,PIBn_std,'b*--');
    xlabel('Strehl Ratio');
    ylabel('Normalized PIB');
    hold on;
    LinearFit(gcf,SRavg,PIBn_avg,12,'r-');
    title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
    drawnow;
    if (ppt) ToPPT(); end;

    nf([.8 .8]);
    errorbarxy(SRavg,PIBn_avg,SRstd,PIBn_std);
    title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
    xlabel('Strehl Ratio');
    ylabel('Normalized PIB');
    drawnow;
    if (ppt) ToPPT(); end;

    nf([.8 .8]);
    plot(SR(:),PIBn(:),'b.'); hold on; LinearFit(gcf,SR(:),PIBn(:),12,'r-');
    title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
    xlabel('Strehl Ratio');
    ylabel('Normalized PIB');
    drawnow;
    if (ppt) ToPPT(); end;

    nf([.8 .8]);
    plot(SR(:),PIBn(:),'b.'); hold on; LinearFit(gcf,SR(:),PIBn(:),12,'r-');
    title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
    xlabel('Strehl Ratio');
    ylabel('Normalized PIB');
    drawnow;
    if (ppt) ToPPT(); end;
```


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```
    nf([.8 .8]);
    plot(log(PIBn(:)),log(SR(:)),'b.'); hold on;
    LinearFit(gcf,log(PIBn(:)),log(SR(:)),12,'r-');
    title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
    xlabel('ln(Strehl Ratio)');
    ylabel('ln(Normalized PIB)');
    drawnow;
    if (ppt) ToPPT(); end;

    nf([.8 .8]);
    plot(log(PIBn_avg),log(SRavg),'r*'); %,log(PIBn_std),log(SRstd));
    hold on;
    LinearFit(gcf,log(PIBn_avg),log(SRavg),12,'r-');
    xlabel('ln(Strehl Ratio)');
    ylabel('ln(Normalized PIB)');
    drawnow;
    title(sprintf('Bucket Diameter = %.1f x DL',NxDL));


    %% single axis search
    v = [0.23; 1.1; 0.8; 0.1;0];
    fit = v(3).*(1 - v(2)*exp(-SRavg/v(1))) + v(4);
    err = sqrt( sum( (fit-PIBn_avg).^2));
    vbest = v;
    besterr = err;
    close all;

    h=nf([ 1115         217         782         644]);
    for iter=1:length(v)*3;
        for i1 = 1:length(v);
            i2=1;
            SF = 1.0;
            while(i2<100 && abs(SF)>0.0001)
                i2 = i2 + 1;
                fit = v(3).*(1 - v(2)*exp(-SRavg/v(1))) + v(4);
                lasterr = err;
                err = sqrt( sum( (fit-PIBn_avg).^2));
                if (err<lasterr)
                    v(i1) = v(i1) + SF;
                else
                    SF = SF .* (-0.5);
                    v(i1) = vbest(i1);
                    v(i1) = v(i1) + SF;
                end;
                if (err<besterr)
                    besterr = err;
                    vbest = v;
                    disp('new best');
                end;
                disp(sprintf('iter: %i i1: %i i2:%i SF:%.2e err:%i
vectorValue: %.2e',iter,i1,i2,SF,err,v(i1)));
            end;

            v = vbest;
            fit = v(3).*(1 - v(2)*exp(-SRavg/v(1))) + v(4);
```


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```
            err = sqrt( sum( (fit-PIBn_avg).^2));

            clf;
            errorbarxy(SRavg,PIBn_avg,SRstd,PIBn_std);
            title(sprintf('Bucket Diameter = %.1f x DL; err = %.2e;
%.2e',NxDL,err,besterr));
            hold on;
            plot(SRavg,fit);
            drawnow;
            pause(0.10);
        end;
    end;
    %if (ppt) ToPPT(); end;

    %% final display
    v = vbest; %[0.23; 1.1; 0.8; 0.1;0];
    close all;

    fit = v(3).*(1 - v(2)*exp(-SRavg/v(1))) + v(4);
    err = sqrt( sum( (fit-PIBn_avg).^2))

    h=nf([ 1115         217         782         644]);
    errorbarxy(SRavg,PIBn_avg,SRstd,PIBn_std);
    xlabel('Strehl Ratio');
    ylabel('Normalized PIB');
    title(sprintf('Bucket Diameter = %.1f x DL',NxDL));
    hold on;
    plot(SRavg,fit);
    ax=axis; text(ax(1)+(ax(2)-ax(1))*0.1,ax(3)+(ax(4)-ax(3))*0.1,...
        sprintf('%.2e * (1 - %.2e * exp(-SR / %.2e)) +
%.2e',v(3),v(2),v(1),v(4)));
    drawnow;
    if (ppt) ToPPT(); end;

end;
```


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