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Number of DM Actuators to Represent a Zernike

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1 Introduction

In prior work, we have shown that we can fit Zernike terms with DM influence functions to get an accurate representation of the DM's capability to compensate Zernike aberrations.[1] In this application note, we attempt to derive a more general relationship between the number of actuators and the number of Zernike terms that can be represented leveraging relationships derived for Kolmogorov-spectrum turbulence.

2 Derivation

In prior work, Noll derived an expression for the fitting error of a Kolmogorov-spectrum turbulence screen with a varying number of Zernike polynomials.[2] Hudgin derived an expression for the fitting error of a Kolmogorov-spectrum turbulence screen with a DM with a given actuator spacing.[2] By setting these two fitting error expressions equal to each other, we can derive a relationship between the actuator spacing and the number of Zernike terms to achieve the same fitting error. We can then estimate the number of actuators as $N_{act} = \frac{\pi}{4}\left(N_{act1D}\right)^2$, where N_{act1D} is the number of actuators across the DM diameter (D_{DM}/r_s, where r_s is the actuator spacing). The following is a derivation of an expression for the number of actuators required to approximate a given number of Zernike terms, N_m, where κ is a DM influence function shape fitting term that varies based on the DM influence function shape. For generic plate-type DMs, the number of actuators is approximately equal to the number of Zernike terms.

[1] J. Mansell, "Application Note 7: Using a Membrane DM to Generate Zernike Modes", www.aos-llc.com.
[2] R. Tyson, "Principles of Adaptive Optics: 4th Edition", CRC Press 2016.

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$$\sigma^2{}_{fit} = \kappa\left(\frac{r_s}{r_0}\right)^{5/3}, \sigma^2{}_{fit} = .2944 N_m{}^{-\sqrt{3}/2}\left(\frac{D}{r_0}\right)^{5/3}$$

$$\kappa\left(\frac{D}{r_s}\right)^{-5/3} = .2944 N_m{}^{-\sqrt{3}/2}$$

$$N_{act} \approx \frac{\pi}{4}\left(\frac{D}{r_s}\right)^2 \rightarrow \left(\frac{D}{r_s}\right) = \sqrt{\frac{4N_{act}}{\pi}}$$

$$N_{act} = \frac{\pi}{4}\left(\frac{.2944}{\kappa} N_m{}^{-\sqrt{3}/2}\right)^{-6/5}$$

$$N_{act} = \frac{\pi}{4}\left(\frac{.2944}{\kappa}\right)^{-6/5} N_m{}^{3\sqrt{3}/5}$$

$$3\sqrt{3}/5 \approx 1$$

$$N_{act} \approx 0.97 \cdot N_m \text{ for } \kappa = 0.35 \left(\text{Generic DM}\right)$$
$$\approx 0.8 \cdot N_m \text{ for } \kappa = 0.31 \left(\text{AOS Plate DM}\right)$$
$$\approx 1.48 \cdot N_m \text{ for } \kappa = 0.50$$
$$\approx 4.49 \cdot N_m \text{ for } \kappa = 1.26 \left(piston\right)$$

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